The instantaneous phase of a broadband signal is the instantaneous phase of the sum of its sinusoidal components

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Abstract

The phase of a broadband signal is shown to be the phase of the resultant of sinusoidal components using a Fourier representation. This analytic result justifies the use of broadband phase as a measure of synchronisation between multichannel data sources such as those measured in electroencephalography and magnetoencephalography studies. The theorem brings out a feature of a complex field such as those used in quantum mechanics. It is possible to synchronise multiple components to a common phase. This suggests an avenue into researching the expulsion of undesirable biological compounds from a body through quantum tunneling. Also, broadband phase has been used in a pilot meditation study to demonstrate phase resynchronisation.

1 Introduction

Electroencephalography, *inter alia*, requires the analysis, both spatially and temporally, of multichannel data streams into interdependent sources. One measure of interdependence is pairwise instantaneous phase coupling between signals (Tass et al., 1998; Rosenblum, Pikovsky, and Kurths, 1997). The instantaneous phase of a signal can be determined *via* the Hilbert transform (Le Van Quyen et al., 2001; Gabor, 1946). A suitable Bayesian validation measure based on information theoretic entropy is that of the mutual information present between various aspects of signals such as phase, frequency, and amplitude (Le Van Quyen et al., 2001; Shannon, 1948; Barnard and Bayes, 1958). Finite time series can be approximated to an arbitrary degree by Fourier series. Filtering a time series will weight the sinusoidal components. A bandpass filter eliminates all frequencies outside a set range. Any bandpass-filtered signal larger than a few Hertz is termed broadband — there are multiple components.Boashash (1992) warns against interpreting the instantaneous frequency of such signals.

An analytic signal is a complex quantity with the original signal as the real component and the Hilbert transform-derived $\frac{\pi}{2}$ phase-shifted signal as the imaginary component. From the analytic signal the instantaneous phase and amplitude can be calculated. Here we validate the interpretation of the instantaneous phase of such a signal as that of the sum of the individual sinusoidal components.

2 Derivation

Let $s\ t$ denote a time series. The Hilbert transform, $\hat{s}\ t = \mathcal{H}\ s$, of the real-valued signal, $s\ t$, can be used to generate an analytic signal, $s_A\ t = s\ t + i\hat{s}\ t$.

$$\mathcal{H}\ s \quad = \quad (h * s)\ t = \int_{-\infty}^{\infty} (s\ \tau)(h\ (t-\tau))d\tau \tag{1}$$

$$\tag{2}$$

where $h\ t = \frac{1}{\pi t}$, the impulse response of a Hilbert filter, and $*$ is the convolution operator. The Hilbert filter reflects the properties of a potential in a conservative radial field such as the electromagnetic, represented by the complex field. Leading to

$$\mathcal{H}\ s \quad = \quad \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{s\ \tau}{t-\tau}d\tau. \tag{3}$$

$$\tag{4}$$

The analytic signal can be cast in terms of amplitude and phase $s_A\ t = (A\ t)e^{i(\phi\ t)}$, $A \in \mathsf{C}, \phi \in (-\pi, \pi]$, which leaves the task of extracting the time-dependent phase of a broadband signal. Unfortunately, there is degeneracy in the combination of time-dependent phase and amplitude. However, a Fourier series approximation removes this difficulty.

Theorem 2.1 (Broadband Phase)**.** *The instantaneous phase, $\phi\ t$, of a broadband signal, $s\ t$, is the instantaneous phase of the sum of its sinusoidal components.*

Proof. Let $d\ t$ be a Fourier series representation of a discrete sample of $s\ t$ with $2N+1$ samples ($t \in [0, 2N]$),

$$d\ t = \sum_{n=0}^{2N} a_n \cos n\frac{\pi}{N}t + b_n \sin n\frac{\pi}{N}t, \quad a_n, b_n \in \mathsf{R}.$$

Taking the Hilbert transform of $d\ t$ and bearing in mind that, since $\frac{\sin\tau}{\tau}$ and $\frac{\cos\tau}{\tau}$ are Riemman-integrable, the integral over a summation is a summation over integrals, $\int(f+g) = \int f + \int g$,

$$\hat{d}\ t = \mathcal{H}\ d \quad = \quad \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\sum_{n=0}^{2N}(a_n \cos n\frac{\pi}{N}\tau + b_n \sin n\frac{\pi}{N}\tau)}{t-\tau}d\tau \tag{5}$$

$$\quad = \quad \frac{1}{\pi}\sum_{n=0}^{2N}\int_{-\infty}^{\infty}\frac{a_n \cos n\frac{\pi}{N}\tau + b_n \sin n\frac{\pi}{N}\tau}{t-\tau}d\tau \tag{6}$$

$$\tag{7}$$

The integration proceeds by re-expressing the sinusoids as exponentials,

$$\cos\tau = \frac{\exp i\tau + \exp -i\tau}{2},\ \sin\tau = \frac{\exp i\tau - \exp -i\tau}{2}.$$

There is a question of convergence. A discontinuity, or singularity, exists at $\tau = t$, but since $\exp i\tau$ has no discontinuities and is bounded, the convolution integral exists and has no singularities. Integration by parts yields,

$$= \sum_{n=0}^{2N} (a_n i \sin\frac{n}{N}\pi t - b_n i \cos\frac{n}{N}\pi t),$$

so that

$$d_A\ t = (d + \hat{d})\ t = \sum_{n=0}^{2N} a_n(\cos\frac{n}{N}\pi t + i\sin\frac{n}{N}\pi t) \quad (8)$$

$$+b_n(\sin\frac{n}{N}\pi t - i\cos\frac{n}{N}\pi t) \quad (9)$$

$$= \sum_{n=0}^{2N} (a_n + i(-b_n)) \exp i\frac{n}{N}\pi t \quad (10)$$

$$= \sum_{n=0}^{2N} c_n \exp i\phi_n \exp i\frac{n}{N}\pi t, \quad c_n \in \mathsf{R}, \phi \in (-\pi, \pi]. \quad (11)$$

$$(12)$$

Filtering is the application of a weighting function to the coefficients, c_n, and bandpass filtering selects a specific interval, $[l, h] \subseteq [0, 2N]$. The phase of the analytic signal, $d_A\ t$, is the argument

$$\arg d_A\ t = \arctan\frac{\Im\ d_A}{\Re\ d_A} = \arctan\frac{\hat{d}\ t}{d\ t} \quad (13)$$

$$= \arctan\frac{\sum_{n=0}^{2N} c_n \sin\phi_n \sin\frac{n}{N}\pi t}{\sum_{n=0}^{2N} c_n \cos\phi_n \cos\frac{n}{N}\pi t}, \quad c_n \in \mathsf{R}, \phi \in (-\pi, \pi] \quad (14)$$

$$= \phi\ t, \quad (15)$$

$$(16)$$

which is the angle of the phasor resulting from summing each single frequency sinusoidal component. □

Note that the phase is not the sum of component phases, but the phase of the *resultant* vector, which is arrived at through time-independent scaling, c_n, initial phase offset, ϕ_n, and simple vector addition of each sinusoidal component. This is in contrast to Pockett, Bold, and Freeman (2009) who claim, by visual inspection, that the broadband phase is the sum of the resultant phases. Those authors could also have achieved more robust conclusions by using surrogate data and mutual information measures to eliminate the question of common references and the use of pink noise as a control. Note that the use of mutual information can pick up both temporal and spatial separations.

An arbitrary degree of precision up to the smallest energy level can be achieved. Thus a broadband signal has a single well-defined phase, a reflection of the interdependence of dimensions in a complex quantity. While the Hilbert filter is acausal, and cannot be used online, it has been used here to reconstruct the analytic signal. Quantum state evolution is already analytic. This can be related to an action-angle interpretation of phase space configurations in quantum mechanics. Note that the steady phase evolution is compensated for by modulations in amplitude. This suggests an avenue for explorations into teleportation of biological entities. Take a biological molecule with integer spin. If the phase evolution is synchronised, then there is a single overall phase and so quantum tunneling through barriers of disparate frequencies becomes like moving through low energy walls.

Sharma et al. (2007) conducted a *panchakarma* study that involved meditation and other techniques to alter the metabolome of a study group. Synchronisation of energy states in the brain through meditation might, through *hatha* yoga practice, harmonise mental energy states with harmonised centres throughout the bodily organs, leading to the ridding of disharmonious compounds.

The question of how to synchronise these phases is vexing, but an unpublished dataset of mine analysed with the `hsignal` package (McPhail, 2010) examining mental resets in a concentration meditation protocol appears to produce phase slips, which can be interpreted as resynchronisation events. The protocol was simply to meditate and at each point that the subject notices a thought arising a single button is pressed. Phase slips and resynchronisation occur after the button press and thus can not be attributed to the readiness potential. Unfortunately the sample size was one and can only be treated as anecdotal evidence. Pockett, Bold, and Freeman (2009) used averaged epochs and found a 100 ms range of increasing synchrony. If the resynchronisation events are caused by discharges of energy storage driven by the approximately 55 Hz oscillations (often masked by or filtered with by a notch filter because of mains electricity frequency) of the Na+/K+ transmembrane protein behind dendritic currents then an averaging of epochs will miss the phenomenom (Crick and Koch, 1990; Hameroff, 1998; McPhail, 2009; Buzsáki, Anastassiou, and Koch, 2012). These phase slips do not necessarily occur at the median frequency of the broadband filter.

3 Conclusion

I have outlined a foundation for the sound analysis of spatiotemporal datastreams. The phase of a broadband signal is the phase of the sum of the contributory phasors. There is scope for application of this result to the verification of the putative ability of Vedic yogis to dispel disharmonious compounds from their bodies through meditation and energy harmonisation.

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