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A Bose-Einstein condensate is a Bose condensate in the laboratory ground state

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Bose-Einstein condensates of weakly interacting, ultra-cold atoms have become a workhorse for exploring quantum effects on atomic motion, but does this condensate need to be in the ground state of the system? Researchers often perform transformations so that their Hamiltonians are easier to analyse. However, changing Hamiltonians can require an energy shift. We show that transforming into a rotating or oscillating frame of reference of a Bose condensate does not then satisfy Einstein's requirement that a condensate exists in the zero kinetic energy state. We show that Bose condensation can occur above the ground state and at room temperature, referring to recent literature.

> 'I assert that in this case a steadily growing number of molecules compared to the total density will go over into the 1^{st} quantum state (state without kinetic energy), while the remainder of the molecules will distribute themselves according to the parameter value $\lambda = 1 \ldots$ …one part "condenses," the rest remains a saturated ideal gas.' [1, p. 418]

1. Introduction

Stamper-Kurn *et al.* [2] found hydrodynamic excitations and co-oscillations between a Bose-Einstein condensate and the accompanying thermal cloud. Rotating a Bose condensate, which looks stationary in a rotating reference frame, can create excited vortices [3,4]. The dynamics of fractional vortices were explained by Ji *et al.* [5].

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Matter waves, such as dark [6] and bright [7] solitons, and rogue waves [8], described as metastable states, are evidence of non-ground state configurations, but these are treated as perturbations to an underlying ground state [9]. Fabbri *et al.* [10] found excitation resonances on one-dimensional lattices.

In Brown *et al.* [11] we showed that an entire Bose condensate formed in a non-zero momentum state, as opposed to some components being in a metastable state as in the experiments with solitons and vortices. We used a Bragg pulse to excite 50% of a population of Rubidium-87 atoms in the ground state of an approximately harmonic trap into the $|2\hbar k\rangle$ momentum state. The higher momentum state oscillated in the trap at the trap frequency and collided with the zero momentum state, scattering atoms. These atoms then coalesced back into a condensate at the average momentum, the centre of mass, state, $|1\hbar k\rangle$, through Bose-enhanced stimulation [12]. This centre of mass state was originally empty. We showed that the temperature of this state was higher than the ground state, but still remained below the critical temperature, T_c. We also found that the new condensate was sometimes multiply seeded, which caused gray solitons through the Kibble-Zurek mechanism. This is evidence that condensates can occur above the ground state energy level.

In the literature, a Bose-Einstein condensate is described as an ensemble of interacting bosons that macroscopically occupy the *ground state* of a system. Here we demonstrate that the transformation from the $|0\hbar k\rangle$ momentum state of a simple harmonic oscillator into the $|1\hbar k\rangle$ momentum state is an energy-requiring transformation that shows that our Bose condensate was not in the ground state of the system. It is also not an inertial frame of reference. After that, we contrast an oscillating frame with transformations in a system of circular rotations. These too require energy, but the frames of reference remain inertial. We then point out that the laboratory ground state still has non-zero kinetic energy. Finally, we justify the existence of quasi-particle Bose condensates at room temperature.

2. Simple Harmonic Oscillator

Starting from a simple harmonic oscillator reference frame centred at the $|0\hbar k\rangle$ state, we shall show that transforming into the frame oscillating at the $|1\hbar k\rangle$ rate requires energy. Then we show that the frame of reference is not inertial.

(a) Transformation to $|1\hbar k\rangle$ momentum state

The $|1\hbar k\rangle$ state has some energy, $E_1 = \hbar\omega$, which can be determined by finding the energy eigenvalues of the Hamiltonian. We want to transform into the frame of reference where the $|1\hbar k\rangle$ state is stationary.

From the simple harmonic oscillator, in action-angle coordinates, we know that

$$P = \frac{E}{\omega} = \frac{\hbar n\omega}{\omega} \tag{2.1}$$

$$Q = \omega t + \alpha \tag{2.2}$$

and we want to transform into a new frame of reference

$$P' = P - \frac{E_1}{\omega} \tag{2.3}$$

$$Q' = Q. \tag{2.4}$$

The old and new Hamiltonians are

$$H = \omega P \quad (2.5)$$

$$H' = \omega P' \quad (2.6)$$

$$= \omega P - \omega \frac{E_1}{\omega}. \quad (2.7)$$

The difference between the Hamiltonians is

$$H' - H = -E_1, \quad (2.8)$$

which is non-zero and thus shifts the energy of the Hamiltonian. In the oscillating reference frame, the system appears to have $E_1 = \hbar\omega$ less energy than when we consider the oscillation. The ground state still has less energy than the $|1\hbar k\rangle$ state, even though the higher energy state appears stationary in a transformed picture.

(b) Frames of Reference

A Galilean, or inertial, frame of reference is one in which the reference frame is not accelerating. A Galilean transform is a transformation between coordinate systems that can be achieved through a combination of uniform motion, translations, and rotations. These are all linear operations that can be written as a matrix algebra.

The canonical transformation into the $|1\hbar k\rangle$ state is not a linear transformation. In the oscillating frame of reference there is a non-zero acceleration, thus the frame of reference is not inertial.

We can rewrite the oscillator Hamiltonian in terms of kinetic and potential energy,

$$H = \frac{p^2}{2m} + V(q), \quad (2.9)$$

where $V(q) = \frac{m\omega^2}{2} q^2$. Since this is a conservative field, we can calculate the force at position q as $F(q) = -m\omega^2 q$, and from Newtown's second law we have $a = -\omega^2 q$. Thus there is a non-zero acceleration, the value of which depends on the particle's position. Clearly, this is not an inertial frame of reference.

3. Circular Motion

Contrast an oscillating body, which acquires and loses potential energy, with circular motion. In quantum mechanics, considering only orbital angular momentum, we can generate a time-dependent rotation about an axis, z, with the rotation operator,

$$R(z, \omega) = \exp(i\omega t L_z). \quad (3.1)$$

Our transformed wave function is

$$\psi'(t) = \exp(i\omega t L_z)\psi(t), \quad (3.2)$$

and the Schrödinger equation is

$$i\frac{d}{dt}\psi'(t) = (\omega L_z \exp(-i\omega t L_z) + \exp(i\omega t L_z) H(t) \exp(-i\omega t L_z))\psi'(t). \quad (3.3)$$

The transformed Hamiltonian is

$$H'(t) = \omega L_z \exp(-i\omega t L_z) + \exp(i\omega t L_z) H(t) \exp(-i\omega t L_z). \quad (3.4)$$

If we look at a half period of rotation, from $t : 0 \to \pi$, the exponentials will equal 1 and -1, and thus, in a conservative field with rotational symmetry, we have the energy difference between the Hamiltonians,

$$H'(t) - H(t) = -2\omega L_z, \tag{3.5}$$

which is non-zero and thus shifts the energy of the Hamiltonian. In the rotating reference frame, the system appears to have $2\omega L_z$ less energy than when we consider the rotation.

The shift to the rotating frame of reference is an energy requiring transformation. This implies that the lowest energy state of a trapping system in a laboratory frame rotating about the Earth actually has kinetic energy. Also, a condensate rotating in the laboratory frame will have non-zero kinetic energy. Becker *et al.* [13], using an experiment aboard a rocket, performed condensation and interferometry tests during launch and generated a condensate in Earth orbit, which involves a non-inertial frame of reference during launch and non-zero kinetic energy with respect to the Earth's frame of reference during their six minutes of microgravity. So while a Bose-Einstein condensate might have kinetic energy, it is non-the-less in a ground state. The difference between circular motion and oscillatory motion is that the rotating frame of reference is inertial, whereas the oscillating frame of reference has non-zero acceleration. Thus we have supported the experimental evidence that Bose condensates can form in non-ground states and in non-inertial frames of reference.

4. The Many Body Case

In the treatment of circular motion, we can already extend to the many body case, as the rotation operator simply acts upon each particle. The oscillator Hamiltonian was constructed for the single particle case. In the case of many interacting particles, the treatment still holds. It is true that a many particle system will have interaction terms in the Hamiltonian, such as the probability density term in the Gross-Pitaevskii equation, however we can use a canonical transform between Hamiltonians, and a legal transform will operate on the interaction terms suitably. The interactions are dependent on the relative positions of particles rather than their centre of mass motion, so that in a non-relativistic setting, will not be affected by a transformation. A many body Hamiltonian, transformed to a new frame of reference, will still be subject to the same energetic requirements.

Perturbations to an underlying ground state, such as with solitons, vortices, and rogue waves, arise because in experiments the atoms have non-zero interaction strengths. These non-zero couplings imply that there will be some non-zero energy as a result of the non-vanishing proximity of adjacent particles. This uneven distribution of energy has been analysed both hydrodynamically and as quasi-particle excitations [14]. In a theoretical analysis of condensates with attractive interactions, it was found that a build-up of energy at the core of the condensate was dissipated as a rogue wave. An example of a collective oscillation that arises from the phase coherence brought about by interparticle interaction. The presence of these couplings greatly affects the condensate regime [15]. If there is a phase matching requirement for transitions between adjacent points in phase space, a collective oscillation might enhance the occupancy rate of a mesoscopic state.

Another level of complication comes about when different species of particle are introduced, such as with spinor condensates, metastable states of which can theoretically give rise to solitons [16]. While these features are evidence of non-zero kinetic energy, they can be treated as perturbations to the potential ground state. The experiment we have described is an observation of the formation of an entire condensate in a higher momentum state.

5. Quasi-particle Condensates

We have shown that Bose condensates can form in a state with kinetic energy. Miesner *et al.* [12] found a rate equation encapsulating Bose-enhanced stimulation derivable from spin statistics [17]. Fröhlich [18] conjectured that Bose condensation could occur in biological systems, but he may not have identified the correct dipole molecules [19]. This form of condensation might be relevant to quantum biology [20]. Fröhlich's treatment used an effective ground state to find a small frequency range of longitudinal energy storage. His rate equations were subsequently shown to be equivalent to a second quantised form [21] that is like three-wave mixing in non-linear optics.

Recalling that mass and temperature are two proportional factors in the de Broglie wave equation,

$$\lambda_{DB} = \frac{h}{\sqrt{2\pi m k_B T}}, \tag{5.1}$$

so that for Rubidium-87 of mass about 10^{-27} kg with a critical temperature of, say, 10^{-7} K, the ratio is about 10^{-34}. Thus, for the de Broglie wavelength to be the same at room temperature $(300\ K)$, we require a mass less than about 10^{-37} kg. Also, a further decrease in mass of the quasi-particle by 10^6 will increase the de Broglie wavelength by 10^3. Small effective mass quasi-particles, like magnons [22], photon-dye interactions in a cavity [23], and non-linear photon interactions in a erbium-ytterbium co-doped fibre cavity [24], Bose condense at room temperature. This helps explain high temperature superconductivity [25], a maximum temperature for which might be calculated from the effective mass of Cooper pairs and the lattice spacing. While lasers are coherent, photons in a vacuum cannot interact and condense. This requires the intermediary non-linear matter interaction that overcomes dispersive forces, such as that in a laser cavity.

6. Conclusion

We have shown that the transformation into the oscillating $|1\hbar k\rangle$ reference frame requires energy and thus there is the formation of a Bose condensate that is not in the ground state of the trapping system. We have also shown that transforming into a rotating frame of reference requires energy and so an apparatus ground state can still have overall kinetic energy. Thus we have justified that Bose condensates can be created in non-ground states and non-inertial reference frames. Bose condensates attract particles into a macroscopically occupied state, not necessarily the ground state, through Bose-enhanced stimulation. From the perspective of quasi-particles, we have shown that Bose condensation can occur at room temperature.

Ethics. No ethics approval was required for this research.

Authors' Contributions. A. V. H. M. conceived of and wrote the paper. M. D. H. supervised and asked for a treatment of circular motion.

Competing Interests. The authors declare no competing interests.

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